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First principles in mathematics as data and as vincula: A critique of Thomas Reid by Dugald Stewart[1]

Claire Etchegaray

The Scottish Common Sense School was keen to draw an analogy between mathematics as a system and the general logics of the mind. It did so in order to understand what the *evidence* of judgment and of reasoning consist in. That is a feature by which, according to Richard Olson, the Common Sense School 'diverged from its Baconian foundations to adopt an almost Cartesian stance',[2] presumably under the influence of 'the great emphasis placed on the axiomatic basis of mathematics by Euclid' brought to the fore through the English translation of the *Elements* by Robert Simson (published in 1756) as well as his works on Greek geometry. Thus, at the beginning of the *First Essay on the Intellectual Powers of Man*, Reid credited mathematicians for 'having had the wisdom to define accurately the terms they use, and to lay down, as axioms, the first principles on which their reasoning is grounded".[3] He wished to do the same in the philosophy of the mind, by clarifying basic terms and laying down the proper principles in the many different domains of reasoning. Given that the philosophy of common sense aimed at accounting for mental operations in each domain, one of the issues it had to address was mathematical reasoning. Thus, a leading thread can be noticed, that goes from mathematics (especially Euclid's *Elements*) to common sense principles, and then from common sense psychology and the logics of the mind to mathematical reasoning again.

Dugald Stewart was well aware of the issues that derived from this give-and-take, and he devoted quite a lot of work to understanding its merits and its limits. According to him, the theory of mathematical reasoning which was a part of a philosophy of the mind, the latter being inspired by a mathematical

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[2] Richard Olson, *Scottish Philosophy and British Physics, 1750 – 1880: A Study in the Foundations of the Victorian Scientific Style*, ch. 3 (Princeton, 1975), 55.

[3] D. Brookes (ed), Thomas Reid, *Essays on the Intellectual Powers of Man* (Edinburgh, 2002), First Essay, chapter 1, 17. In this article, the reference will be abridged in the following way : *IP*, I.1 B 17.

model, should have been more cautious in the conception of 'principles' and 'axioms'. It is in this context that he developed an original and searching critique of Reid's thought.[4]

Stewart identifies an ambiguity in the Reidean concept of 'first principles' which, according to him, leads to an unsatisfactory account of mathematical evidence. In doing so, he brings forward the issue of foundations in mathematics in a very different way than Reid's, and opens the way for further considerations on systematical axiomatisation later in the nineteenth century. First principles were indeed for Reid the source of evidence of the judgment. He focused on the question of mathematical foundation by analyzing the *evidence* upon which reasoning is founded: in other words, he was interested in the warrant of mathematical assent. With Stewart, we move from this question to the following one: how must the body of mathematical science be framed? Stewart focuses on systematical foundation and not only on psychological foundation, because he requires a distinction between first principles as *elemental truth* which are taken for granted, but not sufficient to infer some specific conclusions, and first principles as *first data* which have to define the objects of the subsequent reasonings.

This critique is but one of the pieces of Stewart's general attack against the widespread view that the principle of identity is the only foundation of mathematics. Dealing with mathematical demonstration, in the second volume of the *Elements*, Stewart indeed criticizes the theory, which according to him is commonly received since Leibniz, that 'all mathematical evidence ultimately resolves into the perception of identity'.[5] He thinks that this thesis (I shall call it MI) leads to skepticism in mathematics and consequently, as mathematical evidence was traditionally regarded as the highest kind of evidence, to an even more radical skepticism. If (MI) is right, he argues, then mathematical judgment would be tautologic or nugatory, and mathematical reasoning would fail to make us discover any unknown properties. Stewart

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4 Dugald Stewart's father, Matthew Stewart, was Simson's student and friend. Matthew Stewart was Professor in Mathematics in the University of Edinburgh, although at the end of his career (from 1773 to 1785) a severe illness constrained him to be supplied by his son. So Dugald Stewart's reflexion on mathematics is not from a distance. His background includes the practice of his father, that of his friend John Playfair, as well as his own practice as a teacher in mathematics.

5 *Elements of the Philosophy of the Human Mind*, Second volume (1814), Part II, ch. 2, section 3, article 2 (in Sir William Hamilton (ed.), *The Collected Works of Dugald Stewart* (Edinburgh, 1854), 123. In this article, this reference will be noted : *Elements*, Vol. 2 (1814), II.ii.3–2, 123.

quotes the end of Diderot's *Letter on the Blind*[6] as the paramount of the skeptical argument:

> Put the question to any candid mathematician, and he will acknowledge, that all mathematical propositions are merely identical; and that the numberless volumes written (for example) on the circle, only repeat over a hundred thousand forms, that it is a figure in which all the straight lines drawn from the centre to the circumference are equal.[7]

Reid on mathematical knowledge

Actually, Reid did hold that trifling truth is not qualified to be knowledge. In a Lockean way, he demanded in the Sixth *Essay*, that axioms should be distinguished from trifling propositions. Axioms are characterized by self-evidence, *and* dignity and utility as well, while identical propositions are so 'trifling' and so 'surfeited by truth' that 'no knowledge can be derived from them'. Reid subscribed to Locke's opposition to the view that 'all our knowledge is derived from these two maxims, to wit, whatever is, is; and it is impossible for the same thing to be, and not to be'.[8] Besides, in Reid's view, evidence of reasoning must not be reduced to axiomatic evidence. The latter is the ground of assent to propositions believed as soon as understood; the former is the ground of assent to conclusions drawn from these already known and believed propositions (properly called *reasons* or *premises*). Therefore, according to Reid, the confusion between an unfruitful syllogism and a proper abstract reasoning has to be avoided. A syllogism only develops in an unfruitful way the axiom of necessary logical truth that 'what is affirmed of a whole genus, may be affirmed of all the species and individuals belonging to that *genus*; and that what is denied of the whole genus, may be denied of its species and individuals'.[9] On the contrary, proper abstract reasoning discovers some

[6] 'Letter on the Blind for the Use of Those Who See', *Diderot's Early Philosophical Works*, trans. Margaret Jourdain (New York, 1972).

[7] Denis Diderot, *Letter on the Blind*, quoted by Stewart in *Elements*, vol. 2 (1814), Appendix, Article 1, 407. Cf. *Lettre sur les aveugles* (Paris, 1972; 1749), 124.

[8] *IP*, VI.7 B 521.

[9] *A Brief Account of Aristotle's Logic*, ch. IV, sect. 4, in Alexander Broadie (ed.), *Thomas Reid on Logic, Rhetoric and the Fine Arts* (Edinburgh, 2004), 125. As early as 1753, in an oration delivered in Aberdeen on April, 9, Reid argues that the syllogism is useless in the sciences in general and especially in mathematics. He observes: 'If

new truth because from mathematical conceptions which are 'true and adequate', it deduces some properties inseparable from the nominal essence of the mathematical objects conceived. 'There is nothing belonging to a plane triangle which is not comprehended in this conception of it, or deducible from it by a just reasoning'.[10]

Indeed mathematical truths can be *learned* because the application of this genus-axiom makes us conceive properties which *we did not* conceive before, although they are inseparable from the nominal essence of the object conceived.

As is well known, although Reid attacks the 'way of ideas' regarding judgment and reasoning about existential or contingent things, he admits Locke's theory of abstract reasoning provided that ' ideas' be only acts of conception and not mental objects of conception. Indeed the only real objects of mathematical conceptions (or ideas), according to Reid, are the primary qualities of things: extension, figure, movement (and, we might perhaps add, duration)[11]. More accurately, mathematical conceptions are *universals*, which are formed by abstraction. We perceive such and such extensions, such and such figures; and though they are never perfectly circular or triangular, we are able to form *general conceptions* joined to a general word ('circle', 'triangle', and so on) as its sign. As mathematical judgment is ontologically neutral, its truth depends only on connections between the notions that are implied. Note that Reid accounts for the origin of the idea of *number* in accordance with this thesis. A number is a conception needed to compare conceived durations, extensions, and so on. Because of the quantitative nature of the primary qualities, their *meson* (common measure) is a *metron* (quantitative standard).[12]

In any case, this account of mathematical reasoning implies that at the starting point the mathematician may *not* conceive intuitively of all the properties

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in any section of philosophy, certainly in mathematics, dialectic ought to bring aid and *yet mathematicians, who in everyone's opinion reason in the proper manner, reject the syllogistic pomp and apparatus as a useless hindrance*.' (*First Oration, Speech delivered in the public auditorium of King's College*, Aberdeen, 9 April 1753, in *The Philosophical Orations of Thomas Reid Delivered at Graduation Ceremonies in King's College, Aberdeen* (Carbondale and Edwardsville, 1989), 37.

[10] *IP*, IV.1 B 304.

[11] *IP*, II.17 B 203.

[12] Cf. *IP*, III.3 B 259. Reid also notices that (integer) number is not always sufficient to measure agreement or disagreement between primitive qualities. Reid, like other post-empirical philosophers, is aware of the issues entailed by irrational and imaginary numbers; hence he says (in contrast with Hume) that the agreement is evaluated with *ratio* rather than *units* (*IP*, B 546).

which are nonetheless inseparable from the mathematical conception. We might attempt to solve the paradox in distinguishing three ways of conceiving a mathematical object, for instance, a triangle: (a) the conception of nominal essence that is the definition which *de jure* includes every property of the triangle; (b) the conception which is *de facto* limited by nature (that means: by the nature of the constitution of the human mind) but which is *de jure* the conception that every mathematician *should* have, namely: the clear and distinct notion which is correlative to sound judgment and right reasoning;[13] (c) the *de facto* conception which is relative to individual skills and understandings, and which *cannot* be a standard in Reid's view. The first conception (a) is not a transcendent idea: it is a mental act that the mind should accomplish although the limitations due to human finitude preclude its being done *immediately*. The genus-principle is the means by which we shall be able to have a conception (b) of what is comprehended in the conception (a).[14]

Although Reid attempted to account for the status of mathematical reasoning as an *informative* application of the genus-principle, Stewart thinks that Reid did not save it from skeptical threats, because he made two major mistakes. First, he did not explain clearly the sense of the word 'principle'; second he wrongly held that mathematical evidence was intuitive. The next sections are devoted to these pointss.

Stewart's discussion of the role of the principle of identity

Stewart thinks that past philosophers did not realize that the principle of identity was not sufficient because they did not grasp the distinct meanings of the word 'principle'. Reid in particular entertained the confusion. Past philosophers failed to use properly the meaning of the *principle* of identity, and thence did not pay attention to the systematical requisites of mathematics as

[13] Cf. *IP*, IV.1, B 307.

[14] The standard of truth, according to Reid, is not relative understanding but necessary conception, for objective meanings of the conception of the subject and the conception of attribute have to be compared in order to yeld necessary relations. This is the reason why he opposes the *de facto* principle of conceivability, which he interprets as a principle in the way proper to Hume, for instance in the following text : 'Mathematicians have, in many cases, proved some things to be possible, and others to be impossible; which, without demonstration would not have been believed: Yet I have never found, that any Mathematician has attempted to prove a thing to be possible, because it can be conceived; or impossible, because it cannot be conceived" (*IP*, IV.3 B 330–333).

a body of knowledge. Thus, Stewart inquires into the grounding of (MI) in order to point out the origins of its plausibility, but then objects to them. The origins, and consequently the objections, are twofold.

(1) (MI) can be interpreted as a consequence of the thesis that 'the axioms of Euclid are the *first principles* of all our subsequent reasoning in geometry' (call it AxP), and more generally that axioms are the foundations on which any of the sciences is built – including mathematics. Indeed, Euclidian Axioms or 'Common Notions' (as, for instance, 'the whole is greater than its part' or 'things equal to the same thing are equal to one another') might be considered as identical propositions. The link between (AxP) and (MI) had been sustained by Alexander Campbell who argued that Euclidean axioms are 'all in some respects reducible to this axiom, "whatever is, is"' because they are mere 'particular exemplifications of that axiom'. In this respect, Campbell agreed with Locke's views on axioms that, although an axiom can be enunciated in a general proposition, it is already assented to in a particular instance. Though Campbell did concede that 'if axioms were propositions perfectly identical, it would be impossible to advance a step by their means' because no knowledge can be drawn from any proposition where the predicate is the same as the subject, he assumes

> [W]hen the thing, though in effect coinciding, is considered under a different aspect; when what is single in the subject is divided in the predicate, and conversely; or, when what is a whole in the one is regarded as a part of something else in the other; such propositions lead to the discovery of innumerable and apparently remote relations.[15]

But according to Stewart, (AxP) is wrong, because, these propositions (or Common Notions), which are no less essential in arithmetic than in geometry, do not delineate any domain of objects.

> [T]herefore, to explain in what manner the mind makes a transition, in the case of numbers, from the more simple to the more complicated equations, throws no light whatever on the question, *how* the transition is made, either in arithmetic or in geometry, from what are properly called axioms, to the more remote conclusions in these sciences.[16]

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15 A. Campbell, *Philosophy of Rhetorics*, quoted by Stewart, *Elements*, Vol. 2, II.i.1, 27 – 8.

16 *Elements*, Vol. 2, II.i.1, 29 – 30.

(2) (MI) can also be interpreted as a consequence of another thesis, that 'the geometrical notions of *equality* and of *coincidence* are the same' (call it EC). This time, Stewart concedes that (EC) is a *correct assumption*. But because of the confusion of identity with equality, it was thought that in geometry and in arithmetic the mind always states mere identities. Two reasons lead Stewart to object to the view that identity and equality are synonymous. First, if they were synonymous, some mathematical conclusions would be absurd. Thus, Stewart says, that the area of a circle is equal to the area of a square does not mean that they are identical. This example shows that Stewart takes 'identity' to be an identity between the objects conceived. He does not deny that mathematics conflates 'equivalences' and 'equalities'. In arithmetic in particular, he agrees that the mind performs a mere 'comparison of different expressions of the same quantity'.[17] But – and this is the second reason – even if all mathematical propositions (which all express equalities) could have the form of the proposition of identity a=a, the inference itself could not be reduced to an identical proposition.

> Granted, for the sake of argument, that all mathematical propositions may be represented by the formula a=a, it would not therefore follow, that every step of the reasoning leading to these conclusions, was a proposition of the same nature?[18]

The evidence being the ground of assent to the (alleged 'identical') proposition, is not an identical proposition itself. Even if identical propositions could express mathematical truths, as in arithmetic, they cannot constitute mathematical evidence.

Thence the following questions occur. Firstly, what are the first principles of mathematics according to Stewart, and how could they delineate some specific objects (either in geometry or in mathematics)? Secondly, where does the evidence of mathematical reasoning come from?

The first principles in mathematics

The expression 'first principles' is a legacy of Reid's. As is well known, Reid begins the *Essays on the Intellectual Powers* by listing some principles that every man

[17] *Elements*, Vol. 2, II.i.1, 28.
[18] *Elements*, vol. 2, II.ii.3.2, 129.

ought to 'take for granted' in so far as he is not lunatic[19]. They are mentioned at the beginning of the work, because they are points of minimal agreement between the author and his readers. They constitute both the 'foundation of all reasoning and of all science' and 'the *common sense*' without which any discussion would be impossible. As Reid previously said in the *Inquiry*:

> If there are certain principles, as I think there are, which the constitution of our nature leads us to believe, and which we are under a necessity to take for granted in the common concerns of life, without being to give any reason for them; these are what we call the principle of common sense; and what is manifestly contrary to them, is what we call absurd[20]

As no discussion is possible with the fool, no discussion is possible with the man who sustains absurdities. Here is Reid's strategy against the skeptic: he tries to make the skeptic concede that in his mental operations, he always acknowledges the truth which he denies in words. According to Reid, even the skeptic, as well in his mental acts as in his practical conduct, takes for granted that he is a *self*, that his faculties are not deceptive and that there is an external world. His answer to the skeptic consists in bringing him to admit for himself that he does so. And then, once the skeptic is constrained to become aware that he recognizes *evidence* as a *just ground* of belief, he must admit that the principles *taken for granted* are principles of *truth*, that means principles which he (the skeptic) takes as true. This is the reason why, in the Sixth *Essay*, Reid is prepared to enunciate principles of contingent truth and afterwards principles of necessary truth. In mathematics in particular, these principles are the well-known axioms that 'from the days of Euclid', 'mathematicians have very wisely laid down'.[21]

Notwithstanding, Stewart thinks that this account is not sufficient to understand the mental operation of reasoning – especially those of abstract reasoning – because in his view Reid confused two very distinct meanings of 'principle'. The Latin couple *datum* / *vinculum* is used by Stewart to distinguish them. By *data* in the reasoning, he understands that from which the reasoning proceeds (typically, the starting premises). By *vincula* he means what is required to make an inference (as 'links' uniting the reasoning). *First principles* may be first *data* of reasoning, namely premises, reasons. In this sense a *principle* is an

[19] *IP*, I.2.
[20] *Inq*, II.6 B 33.
[21] *IP*, VI.6 B 491: 'Every one knows there are mathematical axioms'.

'assumption … upon which, as *datum*, a train or reasoning proceeds.'[22] But *first principles* may denote something else, namely the *vincula* (the chains or links) in the reasoning. In this sense, a first principle is what is taken for granted in the exercise of reasoning in order to perform an inference. *Vincula* of human reasoning are 'those *elemental* truths … which are virtually taken for granted or assumed in every step of our reasoning; and without which, although no *consequences* can be directly inferred from them, a train of reasoning would be impossible'.[23] For instance, belief in our own identity, or evidence of memory which Reid holds as principles 'taken for granted' and principles of contingent truths are only *vincula*, and not *data*. They are, according to Stewart, the 'fundamental laws of belief' without which neither judgment nor reasoning about reality would be possible.

In the rest of his work, Stewart calls 'first principles' only the *data*, and 'elemental truths' only the *vincula*. According to Stewart, the first principles (as *data*) in mathematics, are the *hypothetical definitions*, whereas Euclid's Axioms (Common Notions) are the *vincula* or 'elemental truths' of mathematics. The 'Common Notions' are precisely so common that they cannot afford *data* upon which a specific science (about specific objects) may be built. Euclid's Axioms are so universal that they do teach us nothing at all. They would be reduced to the useless 'trifling propositions' pointed out by Locke and Reid, were they not so *necessary*. For mathematical evidence depends on them. But they are not sufficient. Beside them, reasoning needs some *data* to fix what it is about: by way of hypothetical definitions, as we shall see.

> The simple arithmetical equations 2+2 = 4 ; 2+3 = 5, and other elementary propositions of the same sort, are (as was formerly observed) mere *definitions* ; perfectly analogous, in this respect, to those of the beginning of Euclid ; and it is from a few fundamental principles of this sort, or at least from principles which are essentially of the same description, that all the more complicated results in the science are derived.[24]

Now, Stewart's distinction between hypothetical *data* on the one hand, and logical *vincula* on the other hand, is striking not only because it is a point of disagreement between two Common Sense philosophers. It is also a matter of

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22 *Elements*, Vol. 2, II.i.1–2, 36.
23 Ibid., 37.
24 *Elements*, II.ii.3;1, 121.

philosophical significance for it opens the way of assigning contingence to the former, and evidence to the latter.

The first data are hypothetical definitions

Stewart critizes the (Lockean and Reidean) thesis that first principles, as first definitions, must be intuitively certain.[25] For Reid, a 'true and adequate' conception was an intuitive conception of the nominal essence of mathematical objects. Thus, in the Fourth *Essay on the Intellectual Powers*, Reid says that the conception of a plane triangle as 'a plane surface bounded by three right lines' is altogether 'true and adequate'.[26] Although the human mind of the mathematician could not immediately grasp every feature of the nominal essence of the triangle, he has a distinct notion of it if this notion is such that every property of the mathematical object is included in it, at least deductively. So, according to Reid, the mathematician has an immediate conception of some essence, the properties of which he is not completely aware. For Stewart, on the contrary, it is enough to assume some hypothesis as first *data*, provided that they are jointly consistent and do not express any impossibility. Inclusion of properties in the first mathematical definitions is not a standard adapted to provide us with absolutely true definitions: it is only a standard for the correctness of the deduction from hypothetical *data*, namely a standard for conditional truth.[27]

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[25] Ibid,, 113–15.

[26] *IP*, IV.1 B 304.

[27] Stewart found the distinction between absolute truths and conditional truths in Pierre Prevost's *Essais de philosophie* (1804), a Swiss philosopher with whom he regularly corresponded, as some manuscripts in the Library of Geneva attest. Prevost had said that absolute truth is the truth of the reasoning about *facts*, and that *conditional truth* is the truth of pure abstract reasoning. They were nonetheless in dispute about (MI): Prevost sustained that the principle of identity was the foundation of mathematics, although he admits that mathematical propositions are not only tautological, because hypothetical definitions are some determinate instantiations of the principle of identity according to him. Cf. Letter from Prevost to Stewart written on 12 October 1814 (BGE Ms. Suppl. 1067/1, f. 5–6), and remarks from Prevost included in the Appendix of the second volume of the *Elements*, 407–14. Cf. Cl. Etchegaray, K. Haakonssen, D. Schulthess and P. Wood (ed.), "The correspondence of Dugald Stewart, Pierre Prevost and their Circle, 1794-1829" and "The Context of the Stewart-Prevost Correspondence" in History of European Ideas, special issue on Dugald Stewart, forthcoming.

> [I]n mathematics, the propositions which we demonstrate only assert a connexion between certain suppositions and certain consequences. Our reasonings, therefore, in mathematics, are directed to an object essentially different from what we have in view in any other employment of our existences, but to trace the logical filiation of consequences which follow from an assumed *hypothesis*[28].

Significantly, when he analyses some attempts to endow the factual sciences with a demonstrative evidence, for example, to confer on physics, morals, politics, and so on such a demonstrative evidence, he refers to the 'artificial or conventionalist' structure of hypothetico-deductive physics, morals or politics.[29] He concedes that with a set of 'arbitrary definitions' (*sic*) it seems that it might be possible to form a science as certain as geometry if we draw consequences correctly. But those artificial and conventional physics, jurisprudence, and so on, lack the very species of evidence which render their system true, just, good. We must be cautious nonetheless in any assignation to Stewart of some kind on conventionalism in the contemporary sense. In this text, he does not defend any mathematical conventionalism strictly speaking, he only argues that mathematical physics, *more geometrico* politics and deductive ethics are 'artificial or conventional' systems *because* it is only necessary that first definitions express no impossibility and be not inconsistent; so, they might *or might not* fit the facts. So, what are we to understand when Stewart considers that first *data* may be arbitrarily given in mathematics ?

Richard Olson has claimed that like Reid, Stewart 'did not leave mathematicians the same freedom to define mathematical entities and formulate mathematical axioms as did [other philosophers]' because 'for Reid and Stewart the … hypotheses of the mathematician had to be suggested and controlled by experience.'[30] Olson argued that for Stewart, because mathematical concepts are suggested by experience and framed from abstraction and generalization, and though they become afterwards free from any dependence on facts, they can be used in natural physics. Though, this description of the process of formation of mathematical ideas is true, we still think that Stewart is more conventionalist than Olson might believe. Several points support the thesis that Stewart's nominalism entails the assumption that mathematical definitions are *contingent*. As M. D. Eddy has already shown, in the first volume of his

[28] *Elements*, Vol. 2, II.ii.3–1, 114.
[29] Ibid., II.ii.3–1, 115–16.
[30] Richard Olson, *Scottish Philosophy and British Physics, 1750–1880*, chap. 3, 72.

Elements, Stewart thinks in contrast with Reid, that our reasoning depends on signs rather than on conceptions.[31] Certainly, Stewart's criticisms of Reid's conceptualism are more striking in the first volume of the *Elements*. There, in the chapter on 'Abstraction', Stewart says that although Reid was right in denying the existence of universal essences, he was wrong in assuming also the existence of *general* conceptions. Actually, for Stewart an idea is 'the particular quality or qualities in which it [an individual] resembles other individuals of the same class ; and in consequence of which, a generic term is applied to it.'[32] Stewart deplores that Reid neglected the mediation of language. The generic term is only a matter of convention. Indeed, the particular quality to which it is applied is no more essential than another one:

> As all classifications are to a certain degree arbitrary, it does not necessarily follow that it is more essential to its existence as an individual, than various other qualities which we are accustomed to regard as accidental[33]

Resemblances are contingently assigned to different individuals. Classifications are 'to a certain degree arbitrary': they are so, because they do not express a natural or real essence, but not totally so, because they generally depend on the human way of life. In this sense, they are *conventional*.

Up to this point, Stewart does not seem so far away from Reid's and Locke's commitments in relation to nominal essence. But Stewart defends a nominalism which is more achieved than Reid's. And this nominalism entails both the rejection of (MI) and the possibility of a new status for the first data in mathematics. Although the claim of nominalism is less radical in the second and the third volume of the *Elements*, throughout the three volumes Stewart insists on the necessity of the mediation of language for the needs of the mind. In 1814, Stewart points out some mental powers involved in generalization in referring to the unconscious habit of *induction* by which we apply a sign to other similar things. Nonetheless he still denies any power of *general* conception since he says that in the process of demonstration, in geometry for instance, 'we certainly think of nothing but the individual diagram before

31 M. D. Eddy, 'The Medium of Signs: Nominalism, Language, and the Philosophy of Mind in the Early Thought of Dugald Stewart', *Studies in History and Philosophy of Science, part C: Studies in History and Philosophy of Biological and Biomedical Sciences*, 37, 3, 373–93.

32 *Elements*, Vol. 1 (1792), I.iv.2, 175.

33 Ibid.

us';[34] and then, there is a process of generalization, that is *an induction* by which we form the habit to 'consider it [the particular conclusion] as a proposition comprehending an indefinite variety of particular truths'.[35] There are neither *general objects* nor is there *general conception*. There are only *particular conceptions* to which a sign are applied, which can also be applied to other particular conceptions. Obviously, the rejection of the general conception may explain Stewart's reluctance to admit some process of *identification* in mathematics. Generality is not the burden of *one* general conception. It is the feature of one name which could be applied to different particular conceptions. Thus, even in arithmetic, 'names of numbers are nothing else than collectives, by which we are enabled to express ourselves more concisely than could be done by enumerating all the units that they contain'.[36] Equations settle equivalence between signs, not identification of particular conceptions in one general conception. If so, we understand that mathematics are *informative*, because, by the mediation of signs, we assign some new equivalence between individuals (whether they be some particular figures or some particular collections of units) which are not those we had in mind at the starting point.

Now are *mathematical definitions* arbitrary? In the second volume of the *Elements*,[37] Stewart introduces the following specificities of mathematical definitions. First, mathematical definitions are settled in unambiguous words. Their use can be 'proper', because of the limited vocabulary and 'the distinctness of the ideas', whereas in other sciences, words have various meanings and the distinctness of the ideas is not sufficient to establish an existential and realistic assumption. In physics we need to show that the definition we lay down corresponds with the facts. Thus, in mathematics, definitions serve as principles, *data* or outset of the reasoning, while in other sciences, they rather are the *results* of the enquiries. Thence, they may be taken as certain in mathematics, whereas in sciences of facts they remain questionable.

So we might think that mathematics is the only science where definitions are not arbitrary because they are 'proper' and 'perfect'. Yet, actually, this propriety and this perfection is inseparable from the status of arbitrary definitions. First, since mathematical definitions result from contingent abstraction too, we may think that they are conventional in the sense that they depend on the human needs in doing mathematics. Certainly this point

[34] *Elements*, Vol. 2, I.ii.2–1, 90.

[35] Ibid., I.ii.2–1, 90–1.

[36] Ibid., II.i.1–1, 28.

[37] Ibid., II.ii.3–1, 118–19.

is not sufficient to prove that the status of mathematical definitions entails a mathematical conventionalism in Stewart. A feature of 'arbitrary' definitions in mathematics of the nineteenth century is not only that they depend on the needs of the mind, but that their content could be different owing to the mathematician's decision. As we shall see, Stewart does not exclude that this sense of mathematical generic terms can be a matter of *choice* depending on mathematician's needs. Thus, Stewart says that generic terms in mathematics furnish 'an exception' to the imperfection of our definitions because in this science, 'the precise import of its generic terms is fixed and ascertained by the *definitions* which form the basis of all our reasonings, and in which, of consequence, the very possibility of error in our classifications is precluded by the virtual identity of all those hypothetical objects of thought to which the same generic term is applied'.[38]

On the whole, Stewart stands in contradistinction to Reid because although he admits that mathematical truth depends on a relation between ideas, he does assume that they rest neither on 'true and adequate' conceptions formed by the virtue of the constitution of our nature,[39] nor on identification in one general conception of different particular conceptions. For these reasons, the mathematical evidence cannot be resolved into the perception or intuition of identity. Moreover, since the requisites of an appropriate definition are only, (1) that it fixes the sense in an unambiguous way ; (2) that this generic term be applied to virtually identical hypothetical objects of thought, the possibility of merely stipulative definitions remains open. In any case, my aim is not to show that Stewart was a straightforward precursor of axiomatic mathematics, because as we shall see, in some other respects, especially his considerations about *evidence*, his thought was not ready for such an epistemological turn.

The vincula of evidence are axioms

The way Stewart considers the axioms (*vincula*) of mathematical reasoning seems to involve a strong reluctance to an undertaking such as mathematical axiomatisation. Undoubtedly, Stewart thinks that there *are* axioms in mathematics. But, they are of limited utility because they are very universal and involved in our mental operations. Although it can be interesting to enunciate them in order to point out some mistake, there is no need to take pains to

38 Ibid., II.ii. 2–1, 95.

39 Cf. *IP*, IV.1 B 304.

formulate all the accurate propositions which express these *vincula*–because mathematical evidence does not stem from such propositions. In reference to Locke, and in perfect faithfulness to Reid, Stewart says that although an axiom can be enunciated in a general proposition, we already assent to it in particular instances. Thence, the genus-axiom is always assented in the mathematicians' practice, and does not even need to be enunciated. Its enunciation is not a mathematical requirement. It gets naturally applied in abstract reasoning in order to discover some new truth, and in syllogism in a rather fruitless way. When Stewart formulates it, he gives it a nominalistic formulation : 'whatever is true universally of any sign, must also be true of every individual which that sign can be employed to express'.[40] Anyway, it describes a natural operation of the mind which does not depend on any propositional expression. So finally, Stewart inherits Reid's opinion on this point. Reid did not intend to settle some propositional evidence in the list of first principles of truth. He just formulated evidence of our different kind of judgment in some general propositions. Reid avows in the Second *Essay on the Intellectual Powers* that 'evidence is more easily felt than described'.[41] Mathematical evidence in particular rest on the *natural fact* that the negation of tautological principles is unbelievable. Thus, Reid says 'that the rules of demonstrative sciences …have no authority but that of human judgment'.[42] Stewart certainly agrees with Reid on this point. In mathematics the attempt at formalizing logical correctness is far from being Stewart's commitment. The source of evidence cannot be propositional or reduced to an identical proposition because it is naturally involved in our mental operations.

Besides, Stewart argues that logical deduction is not sufficient in mathematics. The mathematician involved in algebraic investigations has to exercise judgment (interpretation) otherwise he might irrelevantly apply conclusions. It is not very clear whether Stewart thinks of some non-mathematical applications (in physics, or in any other science of facts), or of *mathematical applications* themselves. There is some plausibility in favor

[40] In the first volume of the *Elements*, the genus-principle is subservient to radical nominalism: 'the evidence of our conclusions appears immediately from the consideration of the words in which the premises are expressed; without any reference to the things which they denote' (*Elements*, vol.1 (1792), I.iv.1, 177). But as we shall see, Stewart takes pains to distance himself from mere Leibnizian or Condillacian calculus of signs.

[41] *IP*, II.20.

[42] *IP*, VII.4, B 565. Cf . P. Rysiew, 'Reidian Evidence', *Journal of Scottish Philosophy*, Vol. III, 2 (Autumn 2005), 116–17.

of the latter hypothesis. In any case, he believes that besides deduction, mathematicians have the task of interpretation and judgment. They have to attend to the meaning of signs and to limit their conclusions to theses conditions of meaning. The difference between signification and denotation is not explicitly expounded by Stewart although it can be reconstructed on the basis of what he says. *Understanding the meaning* means being able, or having 'in our power', 'to substitute, instead of general terms, some one of the individuals comprehended under them'.[43] But this understanding does not require us actually to do it 'at the moment'.[44] It is sufficient to *have the power* to denote. This is why algebraical art is distinguished from *arithmetical computation* in the first volume of the *Elements*.[45] The commentator M. D. Eddy already stressed the role of judgment in algebra, in contrast with calculus: because the mind has to hold that such or such word (here, the mathematical sign) is representative of particular qualities (here, some *quantities*), it must exert judgment: 'Without this cautious exercise of judgment, in the interpretation of the algebraic language, no dexterity in the use of calculus will be sufficient to preserve us from error'.[46] Stewart concedes that the 'talent for ready and various illustrations' could be useful 'for correcting and limiting our general conclusions'. Twenty-two years later, Stewart does not change his mind. In the second volume of the *Elements*, he opposes both Leibniz's *Ars Combinatoria Characteristica* and the Condillacian project exposed in the *Langue des Calculs*.[47] Condillac indeed assumed in this posthumous work that algebraical reasoning is a model for every reasoning, in so far as in algebra reasoning is performed without any need to know the signification of the *signs*.[48] He shows that such a mechanical way of reasoning is not sufficient to preclude errors.

To sum up, Stewart's distinction between *data* and *vincula* in mathematics attests how much a new way of thinking about principles and axioms allows new interests in the systematical structure of mathematics to arise in the early nineteenth century. Stewart pays attention to the necessity of assuming

43 *Elements*, Vol. 1, I.iv.2, 192.

44 Ibid.

45 cf. *Elements*, Vol. 1, I.iv.5, 204.

46 *Elements*, Vol. 1, I.iv.2, 178.

47 *Elements*, Vol. 2, II.ii.3–2, 131. Cf. Etienne Bonnot, Abbé de Condillac, *La Langue des Calculs* (Lille: 1981 ; 1798).

48 Stewart is aware that this thesis was discussed by the French Ideologues as De Gérando and one of Stewart's friend and correspondent: Pierre Prevost. Cf. Joseph-Marie De Gérando, *Des signes et de l'art de penser considérés dans leurs rapports mutuels* (Paris, 1800) and Pierre Prévost, *Des signes envisagés relativement à leur influence sur la formation des idées* (Paris, 1800), 20.

hypothetical definitions as first *data* in mathematics. Nonetheless he does not defend the mathematical needs for the task of *axiomatisation* as the explicit enunciation of axioms because, for Stewart, axioms are still naturally involved in our mental operations, especially in the mathematical practice of reasoning.

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